The Complement of a Set

Before we can talk about the complement of a set we need to define and understand the Universal set. In any problem we are talking about certain “things”. For example, we might be talking about credit students at the community college during a particular term, or we may be talking about poker hands in a game of Five Card Draw, or we may be talking about lengths of the sides of triangles. Whatever it is, there is some, reasonable, view of all the possible things that we are looking at. For each problem we can define a reasonable, non-frivolous, set that encompasses everything that we could be considering in the problem. If we state such a reasonable, non-frivolous collection, we call it the Universal set for that problem. We should note that it is possible to find and state more than one Universal set for a problem. For example, if we are considering sets of credit students at the community college this term, then certainly, the set of all credit students at the community college this term is one possible Universal set. But so too is the set of all students at the community college this term. The set of all community college students <u>this year</u> is also a possible Universal set. A set that would be frivolous in this context would be the set of all living things that come in contact with the community college this term. In general, for any particular problem, we want a Universal set that encompasses everything that we are examining, but also a set that is reasonably small.

That last desire, for a set that is reasonably small, does not mean that we cannot have the Universal set for a particular problem be infinitely large. For example, if we are looking for solutions to the problems of the form $Ax^2 + Bx + C = 0$, for integer values A, B, and C, where $B^2-4AC \geq 0$, then there are an infinite set of possible answers and we might want to look at the Universal set as the set of all real numbers. In this situation, making the Universal set be the set of all numbers and all animals would not make sense at all.

Now, onto the idea of the complement of a set! If we are working on a problem where the Universal set is given as $U=\{2,3,4,\ldots,12,13,14\}$ and if we are given a set $A=\{4,7,10,14\}$ then the complement of A, written as either A' or A^c, is the set of all things in the Universal set that are not in A. In this case that would be $A'=\{2,3,5,6,8,9,11,12,13\}$. For that same Universal set, if we have $B=\{x|x<10\}$ then $B^c=\{10,11,12,13,14\}$. Or, again, if D={y|y is an even whole number} then $D'=\{3,5,7,9,11,13\}$. Finally, if E={z|z is a prime number} then $E^c=\{4,6,8,9,10,12,14\}$. Notice that because we have our original Universal set defined, in this case $U=\{2,3,4,\ldots,12,13,14\}$ we are not confused by the fact that 23 is also a prime number. The Universal set puts a severe limit on the numbers that we can even consider for the problem. If, on the other hand, someone comes along and changes the Universal set, perhaps to U={x| x is a whole number greater than 1 but less than 25} then E^c becomes $\{4,6,8,9,10,12,14,15,16,18,20,21,22,24\}$.

We should take note that for any set A we have that $(A^c)^c=A$. Also, to be really explicit, when using set-builder notation, we really should say “the set of all things x that are elements of the Universal set such that” and we should write that as $\{x\in U|$, followed by the condition for x. In general practice, this explicit restriction on possible values of x is just assumed.